

Unruh Effect and the Concept of Temperature

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Abstract

Based on a discussion of the concepts of temperature, passivity and efficiency in the framework of quantum field theory, the physical interpretation of the Unruh effect is reviewed.

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The Unruh effect [1], *i.e.* the theoretical assertion that any uniformly accelerated observer in an ambient (inertial) ground state is led to describe his measuring results in terms of a Gibbs ensemble, has been extensively discussed in the literature, cf. the comprehensive reviews [2, 3] and references quoted there. Yet in spite of the fact that almost all of its computational and formal aspects have been considered, it seems that no consensus has been reached about its physical interpretation, cf. for example [4–6]. Does the Unruh effect mean that a thermometer waived around by an accelerated observer in empty space will record a non-zero temperature? And, if not, then what is its physical significance? The ongoing debate originates primarily from the frequently ambiguous usage of thermodynamic concepts (such as temperature) in quantum field theory. It is the aim of this article to outline how these concepts are to be incorporated into the microscopic framework and to review the Unruh effect on this basis. We will describe in meaningful physical terms what the theory actually predicts.

It is appropriate to recall in this context the well known fact that there are two different phenomenological concepts of temperature [7]. The first one enters in the zeroth law: Stationary states of physical systems can be subdivided into classes whose members do not change if brought into thermal contact, they can coexist. This observation is the basis for inventing thermometers, *i.e.* observables whose measured values in a given stationary state allow to determine the particular class to which it belongs. The measured values are then taken as empirical temperature of the state (simply called temperature in the following) and it is this quantity which comes closest to the intuitive idea of temperature. The other, less intuitive notion is based on the second law. According to it it is impossible to gain energy by a cyclic operation which utilizes solely the thermal contact with a single equilibrium (hence stationary) state. Equilibrium states are therefore referred to

as passive. Moreover, the maximum possible efficiency of cyclic operations which utilize the thermal contact with two equilibrium states is of a universal (system independent) nature and can in principle be determined experimentally. It allows to characterize each class of equilibrium states by another, more intrinsic parameter. For the sake of conceptual clarity let us call it Carnot-parameter. Under standard conditions this parameter can be determined by a Carnot process which, by its very definition, uses only thermal contacts and adiabatic changes of states, there may be no external forces. One can then establish a one-to-one correspondence between the temperature and the Carnot-parameter and rename the latter “absolute temperature”. This correspondence is lost, however, in the presence of external forces which affect the efficiency of cyclic processes and thereby the value of the Carnot-parameter. It is apparent that the latter point matters in case of the Unruh effect.

Next, let us outline in appropriate generality how the underlying phenomenological concepts are implemented in the theoretical setting of quantum field theory. Since we will consider states of a given physical system at different temperatures, requiring different Hilbert space representations for their description, the algebraic approach to quantum field theory is appropriate here [8]. There the (localized) measurements and operations which one can perform are described by operators which are elements of some specific Algebra \mathcal{A} . We make use here of the Heisenberg picture, where the states of the system, *i.e.* the (positive, linear and normalized) expectation functionals $\langle \cdot \rangle$ on \mathcal{A} , are time independent and the observables depend on time. Their evolution is given by the solutions of the Heisenberg equation,

$$\frac{d}{dt}A(t) = \delta(A(t)), \quad A(0) = A \in \mathcal{A},$$

where the algebraic generator δ depends on the microscopic interactions and on the motion of the observer¹ and t denotes his proper time. Thus we will have to consider different generators, but may restrict attention to those cases where the generator does not depend explicitly on time.

By comparing expectation values at different times, an observer can identify the stationary states. They satisfy $\langle A(t) \rangle = \langle A \rangle$ for $A \in \mathcal{A}$, $t \in \mathbb{R}$ or, equivalently,

$$\langle \delta(A) \rangle = 0 \quad \text{for } A \in \mathcal{A}. \quad (1)$$

Without changing mean values, the fluctuations of the observables A can be suppressed in these states by taking time averages, $\bar{A}^t \doteq \frac{1}{t} \int_0^t ds A(s)$. A stationary state is said to be extremal if these fluctuations completely disappear in the limit of large t , *i.e.* $\lim_{t \rightarrow \infty} (\langle \bar{A}^t \bar{A}^t \rangle - \langle \bar{A}^t \rangle^2) = 0$. Such a state cannot be decomposed into a mixture of other stationary states. It is of interest in this context that, quite generally, any stationary state is extremal or a mixture of extremal stationary states. Let f be the number of observables $A_1, \dots, A_f \in \mathcal{A}$ needed to

¹In a chosen Hilbert space of states the action of $-i\delta$ can often be described by a commutator with a suitable operator (Hamiltonian, Liouvillian *etc.*). The result of this commutation does not depend on the particular choice of a state space, however, and defines the algebraic generator δ .

discriminate the extremal stationary states. Given such a collection of observables one could use the arrays of mean values $(\langle A_1 \rangle, \dots, \langle A_f \rangle) \in \mathbb{R}^f$, describing certain intensive properties of the states, as their labels. We use here the shorthand notation $\langle \cdot \rangle_\sigma$, $\sigma \in \Sigma^f$, where Σ^f is any suitable index set.

The zeroth law corresponds to the observation that there exist classes of extremal stationary states whose members can coexist if brought into thermal contact. They correspond to subsets $\Sigma_\tau \subset \Sigma^f$ labeled by some real parameter τ , the (empirical) temperature. We do not need to discuss here, how these classes are determined in a given model. But it is clear from the preceding discussion that the observables A_1, \dots, A_f would suffice in order to decide to which class Σ_τ a given state $\langle \cdot \rangle_\sigma$ belongs, *i.e.* to determine its temperature. In order to simplify the discussion, let us assume that there exists a single observable $\theta \in \mathcal{A}$ which likewise allows to determine the class of a stationary state. Thus its mean values coincide in all states of a given class,

$$\langle \theta \rangle_\sigma = \Theta_\tau, \quad \sigma \in \Sigma_\tau, \quad (2)$$

but they differ in states belonging to different classes. Any such observable corresponds to a thermometer and its readings Θ_τ can be interpreted as temperature on a θ -dependent temperature scale.

Not all extremal stationary states are equilibrium states in the sense of the second law. In order to identify the subset of equilibrium (passive) states in the theoretical setting one has to study the effects of cyclic operations on the states. As has been shown by Pusz and Woronowicz [9], the effect of such an operation after one (or several) cycles can be described by the adjoint action of some unitary operator $U \in \mathcal{A}$. A given state $\langle \cdot \rangle_\sigma$ is transformed by this operation into the state $\langle U^* \cdot U \rangle_\sigma$ and the difference between the energy of the state after and before this operation is given by

$$\Delta E(U)_\sigma = -i \langle U^* \delta(U) \rangle_\sigma, \quad U = U^{*-1} \in \mathcal{A}. \quad (3)$$

Since one cannot extract energy from equilibrium states by cyclic operations, these states can be identified in the theoretical framework by the passivity condition $\Delta E(U)_\sigma \geq 0$ for all unitary operators $U \in \mathcal{A}$. For given temperature τ the corresponding passive states correspond to some subset $\Pi_\tau \subseteq \Sigma_\tau$. According to the deep results of Pusz and Woronowicz, any such passive state is either a ground state for the underlying dynamics, or a KMS state (which is the appropriate generalization of the notion of Gibbs state for systems in infinite volume [10]).

The Carnot-parameter, which labels equilibrium states, can be determined in the theoretical setting in several ways. A physically transparent method, established by Sewell [11], cf. also [10], is based on the test of correlation inequalities. There one compares for given passive state $\langle \cdot \rangle_\sigma$, $\sigma \in \Pi_\tau$ and any operator $A \in \mathcal{A}$ which is normalized according to $\langle A^* A \rangle_\sigma = 1$ the expectation values $\langle A A^* \rangle_\sigma$ and $i \langle A^* \delta(A) \rangle_\sigma$. The Carnot-parameter C_σ of state $\langle \cdot \rangle_\sigma$, $\sigma \in \Pi_\tau$ is the unique energy value fixed by the inequalities

$$C_\sigma \ln \langle A A^* \rangle_\sigma \geq i \langle A^* \delta(A) \rangle_\sigma \quad \text{for all normalized } A \in \mathcal{A}, \quad (4)$$

respectively $C_\sigma = 0$ if $\langle A^*A \rangle_\sigma = 0$ for some $A \neq 0$. Thus the Carnot parameter describes a global feature of the respective state. Passive states having different Carnot-parameters in a given frame of reference cannot coexist, *i.e.* they have different temperatures.

For passive states in an inertial frame, where the temperature is everywhere the same within the system, there is a one-to-one correspondence between the Carnot-parameter and the temperature. It is then meaningful to take the Carnot-parameter (which does not depend on the largely arbitrary choice of some thermometer observable) as absolute temperature. But this correspondence is lost for passive states in non-inertial frames, where the temperature may vary within the system. We will illustrate this fact below.

To summarize: Given the dynamics of any observer, described by some generator δ acting on the algebra \mathcal{A} , one can (1) identify states which are stationary, (2) exhibit observables which allow to determine the temperature of states, (3) identify equilibrium states by a condition of passivity and (4) determine the Carnot-parameters of equilibrium states which can be interpreted as absolute temperature in the absence of external forces. Since the corresponding mathematical conditions involve only local operators and the infinitesimal generator δ of time evolution, the theory is consistent with the empirical fact that thermal properties of stationary states can be probed locally and at any given instant of time (replacing time averages by ensemble averages).

Having explained, how the thermodynamical concepts fit into the framework of quantum field theory, let us consider now a concrete example. For simplicity, we take the model of a massless scalar free field in four-dimensional Minkowski space \mathcal{M} . We use the metric $(+, -, -, -)$ and standard units $\hbar = c = 1$ as well as $k = 1$ (Boltzmann constant). Choosing proper coordinates $x = (x_0, \mathbf{x})$ in some given Lorentz frame, the field ϕ and its conjugate π at time $x_0 = 0$ satisfy the standard canonical commutation relations

$$[\phi(\mathbf{x}), \pi(\mathbf{y})] = i \delta(\mathbf{x} - \mathbf{y}) \cdot 1, \quad [\phi(\mathbf{x}), \phi(\mathbf{y})] = [\pi(\mathbf{x}), \pi(\mathbf{y})] = 0.$$

After smearing with test functions, their products and sums generate a (kinematical) algebra \mathcal{A} , whose elements describe localized measurements and operations.

(I) Let us consider first the dynamics of the field, as seen by some inertial observer in the given frame. Its generator δ_0 is fixed by the relations

$$\delta_0(\phi(\mathbf{x})) \doteq \pi(\mathbf{x}), \quad \delta_0(\pi(\mathbf{x})) \doteq \Delta_{\mathbf{x}} \phi(\mathbf{x}),$$

where $\Delta_{\mathbf{x}}$ denotes the Laplacian. The well-known solution of the corresponding Heisenberg equations determine the field $\phi_0(t, \mathbf{x})$ at arbitrary spacetime points $(t, \mathbf{x}) \in \mathcal{M}$, t being the proper time of the observer. It is given by

$$\phi_0(t, \mathbf{x}) = i \int d\mathbf{y} \dot{C}(t, \mathbf{x} - \mathbf{y}) \phi(\mathbf{y}) + i \int d\mathbf{y} C(t, \mathbf{x} - \mathbf{y}) \pi(\mathbf{y}),$$

where C is the commutator function

$$C(x) \doteq (2\pi)^{-3} \int dp \varepsilon(p_0) \delta(p^2) e^{-ixp} = -i(2\pi)^{-1} \varepsilon(x_0) \delta(x_0^2 - \mathbf{x}^2).$$

There exist many extremal stationary states in this model. We restrict attention here to the physically significant family of quasifree states which are fixed by the two-point functions²

$$\langle \phi_0(x) \phi_0(y) \rangle_\sigma \doteq (2\pi)^{-3} \int dp \varepsilon(p_0) \delta(p^2) (1 - e^{-\sigma p})^{-1} e^{-i(x-y)p}, \quad \sigma \in \Sigma^4,$$

where $\Sigma^4 \doteq \{ \sigma \in \mathbb{R}^4 \mid \sigma_0 > |\boldsymbol{\sigma}| \}$. Since these states satisfy the Hadamard condition [12], it is meaningful to proceed from \mathcal{A} to a larger algebra \mathcal{A} containing also composite fields such as the stress energy tensor. In order to distinguish the stationary states one may use the first four components of the tensor field

$$\epsilon_{\mu\nu}(x) \doteq -(1/4) \partial_\mu \partial_\nu (\phi_0(x+z)\phi_0(x-z) - h(x+z, x-z) 1) |_{z=0},$$

where ∂ denotes the gradient with respect to z and h is the Hadamard parametrix for the wave equation [12]; it compensates the leading singularity of the product of fields in the indicated coincidence limit. In the present example this regularization amounts to the familiar Wick ordering on Fock space. The expectation values of this tensor field resulting from the above two-point functions are

$$\langle \epsilon_{\mu\nu}(x) \rangle_\sigma = (\pi^2/90) (4\sigma^\mu \sigma^\nu - \sigma^2 g^{\mu\nu}) (\sigma^2)^{-3}, \quad \sigma \in \Sigma^4.$$

In the Lorentz frame with time direction given by σ , this tensor is diagonal and depends only on σ^2 . Thus given σ_1, σ_2 with $\sigma_1^2 = \sigma_2^2$ the corresponding states differ only by their macroscopic flow velocities which, for an observer in the respective center of mass frame, are equal and have opposite direction. Since parity is an unbroken symmetry in the present model a space reflection has no effect on the thermal properties of states, so the two states can coexist in the sense of the zeroth law, *i.e.* they have the same temperature. The scalar field

$$\theta_0(x) \doteq (\phi_0(x+z)\phi_0(x-z) - h(x+z, x-z) 1) |_{z=0}$$

has the expectation values $\langle \theta_0(x) \rangle_\sigma = (12\sigma^2)^{-1}$. Hence it may be used as a thermometer observable, determining the classes of coexisting states Σ_τ , $\tau \geq 0$ by the condition (at any point $x \in \mathcal{M}$)

$$\langle \theta_0(x) \rangle_\sigma = \tau, \quad \sigma \in \Sigma_\tau \subset \Sigma^4.$$

Since for large σ^2 the corresponding functionals $\langle \cdot \rangle_\sigma$ converge to the vacuum state, fixed by the two-point function

$$\langle \phi_0(x) \phi_0(y) \rangle_\infty = (2\pi)^{-3} \int dp \theta(p_0) \delta(p^2) e^{-i(x-y)p},$$

small values of τ correspond to the idea of “cold” and large ones signify “hot”.

² The reader will recognize that these states are commonly interpreted as equilibrium states of inertial observers. We want to outline here, how this interpretation can be justified on the basis of the preceding discussion.

Next, one has to identify the subset $\Pi_\tau \subset \Sigma_\tau$ corresponding to the passive states for given temperature τ . Here the time direction $e_0 = (1, \mathbf{0})$ of the observer matters. We omit the computations relying on the KMS condition [10] and only state the result: For the given generator δ_0 , the condition of passivity is satisfied by a state $\langle \cdot \rangle_\sigma$, $\sigma \in \Sigma^4$ if and only if σ and e_0 are parallel. Hence $\Pi_\tau = \{\sqrt{1/12\tau} e_0\}$ consists of a single point, *i.e.* there exists only one passive state for each temperature in this model (which does not describe chemical potentials, different phases *etc.*). It is also evident on physical grounds that all other states in Σ_τ are not passive since, making use of their non-zero macroscopic flow velocities, the observer could gain energy from these states by cyclic operations, think of a pinwheel.

It remains to determine the Carnot-parameter for the single state associated with Π_τ . The result is $C_\tau = \sqrt{12\tau}$ which, once again, can be derived from the KMS-condition [10]. Since there are no external forces, one can interpret this parameter as absolute temperature T , thereby establishing the relation $T = \sqrt{12\tau}$ between the temperature scale, based on the zeroth law and our choice of a thermometer observable, and the absolute temperature scale, based on the second law.

Of course, these results are well known in one way or another. The main point, illustrated by this example, is the insight that the physical interpretation of states can be deduced from the theory in a systematic, unbiased manner. We will now discuss the Unruh effect from this perspective.

(II) We consider an observer who is at rest at the point $(0, g^{-1}, 0, 0) \in \mathcal{M}$, experiences a constant acceleration $g > 0$ into the 1-direction and drags along his experimental devices on neighboring orbits.³ These observables are described by elements of the subalgebra $\mathcal{A}_g \subset \mathcal{A}$ which is generated by the field and its conjugate smeared with test functions which have support in the region $x_1 > 0$ of the $x_0 = 0$ plane. For the accelerated observer the (kinematical) observables maintain their inertial interpretation, but their dynamics changes. It is given by the generator δ_g

$$\delta_g(\phi(\mathbf{x})) \doteq g x_1 \pi(\mathbf{x}), \quad \delta_g(\pi(\mathbf{x})) \doteq g x_1 \Delta_{\mathbf{x}} \phi(\mathbf{x}) + g \partial_1 \phi(\mathbf{x}), \quad x_1 > 0.$$

Again, the solutions of the corresponding Heisenberg equations are well known. The resulting field $\phi_g(t, \mathbf{x})$, where t now denotes the proper time of the accelerated observer and $\mathbf{x} = (x_1, x_2, x_3)$ the initial position of the field (Rindler coordinates), is localized at the spacetime point $\mathbf{x}_g(t) \doteq (\text{sh}(gt)x_1, \text{ch}(gt)x_1, x_2, x_3) \in \mathcal{M}$ (inertial coordinates). It can be represented in terms of the inertial field according to $\phi_g(t, \mathbf{x}) = \phi_0(\mathbf{x}_g(t))$. Thus for any point in the wedge shaped region $\mathcal{W} = \{x \in \mathcal{M} : x_1 > |x_0|\}$ (Rindler wedge) one has the equality $\phi_g(x) = \phi_0(x)$. Hence the physical interpretation of the field at a given spacetime point does not depend on the way how it was carried there, *i.e.* on the particular observer.

³Clearly, the observer could depart from any point of the $x_0 = 0$ plane with non-zero velocity and be accelerated into any direction. This particular choice of initial conditions simplifies notation. The motion of the observer and of his experimental devices is adequately described by the Fermi-Walker transport which, from the inertial point of view, is given by Lorentz boosts.

There exist many stationary states also for the accelerated observer. Of particular interest is the one-parameter family of quasifree states which, for given $\varsigma > 0$, are fixed by the two-point function, cf. [13],

$$\langle \phi_g(t, \mathbf{x}) \phi_g(s, \mathbf{y}) \rangle_\varsigma \doteq (2\pi)^{-1} \int d\omega (1 - e^{-\varsigma\omega})^{-1} e^{i\omega(s-t)} \int du C(\mathbf{x}_g(0) - \mathbf{y}_g(u)) e^{-i\omega u},$$

where C is the commutator function given above. Taking averages of the observables in \mathcal{A}_g with regard to the time evolution of the accelerated observer, their fluctuations disappear in all of these states in the limit of asymptotic times. Thus the states are not only stationary but also extremal.

Since the states satisfy the Hadamard condition [15] and the Hadamard parametrix coincides with that in the inertial frame (restricted to \mathcal{W}), the accelerated observer has at his disposal the same composite fields as the inertial observer. In particular, he can make use of the thermometer observable introduced above. Its time evolution is given by $\theta_g(t, \mathbf{x}) = \theta_0(\mathbf{x}_g(t))$, hence $\theta_g(x) = \theta_0(x)$ for $x \in \mathcal{W}$, so its physical interpretation remains unchanged as well. The expectation values of the thermometer observable can be evaluated and are given by

$$\langle \theta_g(t, \mathbf{x}) \rangle_\varsigma = (12x_1^2 g^2)^{-1} (\varsigma^{-2} - (g/2\pi)^2). \quad (5)$$

So the following picture emerges: For parameter values $\varsigma < 2\pi/g$ the thermometer observable displays the same value of the temperature all over any given hyperbolic hypersurface $\{y \in \mathcal{W} : y_1^2 - y_0^2 = x_1^2\}$. If $x_1 > 0$ is made smaller, *i.e.* if the hypersurface is moved closer to the edge of \mathcal{W} (the horizon of the observer) the temperature increases, whereas at large distances it tends to 0. Thus the temperatures measured by the observer on his world line and on neighbouring orbits are different.⁴ The validity of the zeroth law could in principle be tested in this situation by an inertial observer whose worldline is tangent to any given hyperbolic hypersurface and who prepares in his frame an equilibrium state of the corresponding temperature. If, at the meeting point, he takes over the thermometer from the accelerated observer and exposes it to his inertial equilibrium state its readings will not change, according to theory. For parameter values $\varsigma > 2\pi/g$ the thermometer observable displays negative values. So there do not exist inertial equilibrium states which can coexist with the respective stationary states in the preceding sense. In other words, the latter states are only thermally stable in the presence of external accelerating forces. The special case $\varsigma = 2\pi/g$, corresponding to the Unruh scenario, will be discussed below.

All states $\langle \cdot \rangle_\varsigma$ may be regarded as equilibrium states in the sense of the second law. To verify this one has to determine the energy transferred to the states by cyclic operations in the accelerated frame, cf. relation (3). One finds (either by general arguments, making use of the built-in KMS-property of the states [2,10], or

⁴Compare the well known example of a self-gravitating star in equilibrium, where the local temperature also varies through the star [14]. The thermometer observable θ_g registers the analogous effect in the present context.

by explicit computations) that the energy of these states always increases by such operations, *i.e.* the states are passive. Thus the observer cannot take advantage of the temperature differences between the hyperbolic hypersurfaces in \mathcal{W} in order to gain energy. These temperature differences, inducing corresponding pressure differences, serve to stabilize the states by compensating the accelerating forces which depend on the distance from the horizon.

The fact that the states $\langle \cdot \rangle_\zeta$ are passive implies that they comply with another prominent property of equilibrium states: If one couples these states locally to those of some small system (*e.g.* an Unruh–DeWitt detector) by introducing a mild interaction term between the two separate dynamics, one can show that in the limit of large times the small system is driven to a state which is in equilibrium with the large system, cf. for example [16, 17]. More precisely, the small system is passive in this limit and has the same Carnot–parameter as the large system before the coupling. So one can use the small system in order to determine this parameter.

Without coupling the state $\langle \cdot \rangle_\zeta$ to some external system, its Carnot parameter can also be determined as indicated in relation (4). The result is $C_\zeta = \zeta^{-1}$. This parameter determines the relation between the acceleration x_1^{-1} felt by the observables at distance x_1 from the horizon and the temperature $\tau(x_1)$ which has to prevail there in order to attain global equilibrium. Making use of relation (5), one obtains in the present model $C_\zeta = g \sqrt{12 x_1^2 \tau(x_1) + (1/2\pi)^2}$. But it should be noticed that this relation is model dependent; it is different, for example, in massive free field theory. Thus, whereas the Carnot parameter dictates the local conditions for global equilibrium, it contains little information about the local thermal properties of the states by itself. In particular, it may not be regarded as some kind of “global temperature” of the system in the presence of external forces.

Having illustrated the general concepts entering into the thermal interpretation of states in quantum field theory, let us finally discuss the Unruh effect. It presents itself for the special parameter $\zeta = 2\pi/g$. As was discovered in [19, 20], the corresponding functional $\langle \cdot \rangle_{2\pi/g}$ coincides with the Minkowskian vacuum $\langle \cdot \rangle_\infty$ on the algebra \mathcal{A}_g .⁵ In particular,

$$\langle \phi_g(t, \mathbf{x}) \phi_g(s, \mathbf{y}) \rangle_{2\pi/g} = \langle \phi_0(\mathbf{x}_g(t)) \phi_0(\mathbf{y}_g(s)) \rangle_\infty.$$

The generally accepted interpretation of this equality is the assertion that a uniformly accelerated observer in the Minkowskian vacuum perceives this state as an equilibrium state. But we can say more about its specific thermal properties.

It follows from the preceding results that the empirical temperature of the vacuum in the accelerated frame is given by $\langle \theta_g(t, \mathbf{x}) \rangle_{2\pi/g} = 0$ at all points in \mathcal{W} .

⁵For other values of ζ there holds a similar, but somewhat weaker statement [18]: The restriction of any state $\langle \cdot \rangle_\zeta$ to the subalgebra of \mathcal{A}_g , generated by ϕ , π smeared with test functions in any given compact region of the half space $x_1 > 0$, agrees with some state in Fock space. But the latter state depends on the size of this region and, with the exception of the vacuum, there is no state in Fock space for which one has agreement on all of \mathcal{A}_g .

Thus it coincides with the temperature in the inertial frame, contrary to the interpretation of the Unruh effect advocated in [1]. The indisputable and intriguing message of the latter article is the observation that the vacuum is passive also for the accelerated observer. Despite the presence of external forces, the observer cannot extract energy from the state by cyclic operations. In a sense, this feature of the vacuum may be regarded as a kind of super-stability.

It remains to discuss the significance of the Carnot parameter in the accelerated frame. In an inertial frame, a Carnot process operating between temperatures $\tau_2 > \tau_1 \geq 0$ has the efficiency $\eta_0 = 1 - C_{\tau_1}/C_{\tau_2} = 1 - \sqrt{\tau_1/\tau_2}$. Hence if one uses the vacuum as colder reservoir, $\tau_1 = 0$, one could attain in principle the maximal possible efficiency 1. If one considers the analogous experiment in the accelerated frame close to the world line of the observer, one gets for the corresponding efficiency $\eta_g = 1 - C_{\varsigma(\tau_1)}/C_{\varsigma(\tau_2)} = 1 - \sqrt{(12\tau_1 + (g/2\pi)^2)/(12\tau_2 + (g/2\pi)^2)}$. It is strictly smaller than the inertial one and decreases with increasing g . This result indicates that in the accelerated frame only such cyclic processes are possible for the given temperatures τ_1, τ_2 whose efficiency stays well below the ideal one of the Carnot process. In other words, due to the presence of external forces, optimal Carnot processes are impossible in the accelerated frame as a matter of principle. This reduction of efficiency affects also processes involving the vacuum as a thermal reservoir. It finds its expression in the increase of its Carnot parameter.

Whereas in accelerated frames the relation between the local temperature and the Carnot parameter in general depends on the microscopic dynamics, this is not so for vacuum states. Irrespective of the dynamics, their Carnot parameter is given by the universal value $C_0 = g/2\pi$, depending only on the acceleration g . Thinking of theories with several vacuum states it follows that an accelerated observer cannot use pairs of different vacua as thermal reservoirs for energy production. This fact is in conflict with the idea (reviewed in [3]) that, for an accelerated observer, the vacuum is filled with a stream of “particles” carrying thermal energy. For this energy would not only depend on the external forces, causing the acceleration, but also on detailed properties of the particles, such as their masses and their interaction. Hence the ability of the vacuum to coexist in accelerated frames with all other vacuum states would be affected, in conflict with theory. So the vacuum is as “empty” for accelerated observers as for inertial ones; its Carnot parameter merely indicates the external acceleration which acts on the observer and his equipment and narrows his ability to perform cyclic processes.

We emphasize that the preceding computations, performed in an appropriate algebraic framework, are consistent with those done in other settings of quantum field theory, cf. [2,3]. It is only the particular interpretation of the mathematical results, reviewed in those articles, which is at odds with the physical picture emerging from the present discussion. The upshot of our analysis is the insight that, within the present context, the thermal interpretation of states requires a more careful application of the basic theoretical concepts: The notions of equilibrium (passivity) and of efficiency of cyclic processes (entering in the Carnot parameter) depend on the motion of the observer. This finds its formal expression in the

appearance of the generator δ of the dynamics in the corresponding mathematical definitions. In contrast, the concept of empirical temperature, which is based on local kinematical observables allowing to compare different states, is independent of the state of motion of the observer. In the present analysis this found its expression in the equality $\theta_g = \theta_0$ of the accelerated and inertial “thermometers”. These facts have to be observed in a thorough interpretation of the Unruh effect and lead to the present differing conclusions.

Our results suggest that, quite generally, the analysis of the thermal properties of states ought to be based on local (pointlike) observables in the presence of external forces since they maintain their inertial interpretation for accelerated observers. This local point of view was advocated first in [21] in a discussion of thermal properties of non-equilibrium states in Minkowski space. In [22] the framework was extended in order to include also curved backgrounds. The method was illustrated there on the example of observers in de Sitter space. A systematic study and slight revision of this framework was presented in [23] where it was also shown that the global Carnot parameter of passive states imposes in general relations between the acceleration, the curvature and the temperature felt locally by an observer. A comprehensive list of references to other work, where this novel approach was adopted, may also be found in the latter article.

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References

- [1] W.G. Unruh, Phys. Rev. **D14** (1976) 870
- [2] S. Takagi, Prog. Theor. Phys. Suppl. **88** (1986) 1
- [3] L.C.B. Crispino, A. Higuchi and G.E.A. Matsas, Rev. Mod. Pys. **80** (2008) 787
- [4] N.B. Narozhnyi, A.M. Fedotov, B.M. Karakov, V.D. Mur and V.A. Belinsky, Phys. Rev. D **65** (2002) 025004
- [5] G.W. Ford and R.F. O’Connell, Phys.Lett. A350 (2006) 17
- [6] J. Earman, Stud. Hist. Philos. Mod. Phys. **42** (2011) 81
- [7] M. Planck, *Treatise on Thermodynamics*, (Translated from the 7th German edition) Dover Publ. Inc. 1945
- [8] R. Haag, *Local Quantum Physics* , Springer Verlag 1992
- [9] W. Pusz and S.L. Woronowicz, Commun. Math. Phys. **58** (1978) 273

- [10] O. Bratteli and D.W. Robinson, *Operator Algebras and Quantum Statistical Mechanics 2*, Springer Verlag 1981
- [11] G.L. Sewell, Commun. Math. Phys. **55** (1977) 53
- [12] R.M. Wald, *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*, The University of Chicago Press 1994
- [13] R. Haag, H. Narnhofer and U. Stein, Commun. Math. Phys. **94** (1984) 219
- [14] R.C. Tolman, Phys. Rev. **35** (1930) 904
- [15] H. Sahlmann and R. Verch, Commun. Math. Phys. **214** (2000) 705
- [16] A. Kossakowski, A. Frigerio, V. Gorini and M. Verri, Commun. Math. Phys. **57** (1977) 97
- [17] S. De Bievre and M. Merkli, Class. Quant. Grav. **23** (2006) 6525
- [18] R. Verch, Commun. Math. Phys. **160** (1994) 507
- [19] S.A. Fulling, Phys. Rev. **D7** (1973) 2850
- [20] G.L. Sewell, Annals Phys. **141** (1982) 201
- [21] D. Buchholz, I. Ojima and H. Roos, Annals Phys. **297** (2002) 219
- [22] D. Buchholz and J. Schlemmer, Class. Quant. Grav. **24** (2007) F25-F31
- [23] Ch. Solveen, Class. Quant. Grav. **29** (2012) 245015